TOPIC 1

Measurement and Mathematics

How Scientists Study Measurement and Mathematics

Are angles always measured in degrees?

A protractor is a device used to measure the size of an angle in degrees. A protractor that is a semicircle has a range from 0° to 180°, because a circle has 360°. The reason there are 360° in a circle may be related to some ancient calendars using 360 days for a year. Early astronomers noticed that the stars seemed to move $1/360^{th}$ of a circular path each night. Long before there were calculators, it was known that 360 was divisible by every number from 1 to 10 except 7, and that it had a total of 24 factors.

In higher levels of physics and mathematics, angles are often measured in radians. The radian measure of an angle has no dimensions. A full circle is equal to 2π radians, making 1 radian equal to $360°/2\pi$ or approximately 57°.

When using your calculator for angle calculations in this text and on the Regents examination, be sure to confirm that your calculator is in the correct mode. For example, the sine of 20.0 degrees is 0.342 and the sine of 20.0 radians is 0.912.

TOPIC 1

Measurement and Mathematics

Vocabulary

absolute error
accepted value
accurate
constant proportion
dependent variable
derived unit
direct squared proportion
directly proportional
experimental value
extrapolation
force
fundamental unit
independent variable
indirect squared proportion
inversely proportional
line of best fit
mass
mean
percent error
precise
range
scalar
scientific notation
SI prefix
SI system
significant figures
slope
standard deviation
unit
variance
vector

Topic Overview

Note to student: *This topic explains some of the process skills based on Standards 1, 2, 6, and 7 that you will use in the study of physics. These skills will be applied to specific content in later topics. Material in this topic will not be tested as definitions or on a purely mathematical basis. However, these concepts and process skills are testable when they are incorporated into specific physics content based on Standard 4.*

Physics is based on observations and measurements of the physical world. Consequently, scientists have developed tools for measurement and adopted standard conventions for describing natural phenomena. These conventions are reviewed below.

Units

A **unit** is a standard quantity with which other similar quantities can be compared. All measurements must be made with respect to some standard quantity. For example, it makes no sense to say the distance between two cities is 26. Distance must be stated in terms of a standard unit. The distance between the cities might be 26 miles or 26 kilometers.

The SI System

The **SI system** provides standardized units for scientific measurements. All quantities measured by physicists can be expressed in terms of the seven **fundamental units** listed in Table 1-1. **Derived units** are combinations of two or more of the fundamental units and are used to simplify notation. Other systems of units are sometimes used when they are more appropriate because of the size of the quantity being measured.

Table 1-1. Units of Measure			
Kind of Unit	**Quantity Being Measured**	**Name of Unit**	**Symbol**
Fundamental SI	length	meter	m
	mass	kilogram	kg
	time	second	s
	electric current	ampere	A
	temperature	kelvin	K
	amount of substance*	mole	mol
	luminous intensity*	candela	cd
Derived SI	frequency	hertz	Hz
	force	newton	N
	energy, work	joule	J
	quantity of electric charge	coulomb	C
	electric potential, potential difference	volt	V
	power	watt	W
	electrical resistance	ohm	Ω
	resistivity	ohm · meter	Ω · m
Non-SI	length	centimeter	cm
	mass	gram	g
	mass	universal mass unit	u
	time	hour	h
	energy, work	electronvolt	eV
	angle size	degree	°

*These quantities are not treated in this review.

SI Prefixes

SI prefixes are prefixes combined with SI base units to form new units that are larger or smaller than the base units by a multiple or submultiple of 10. The symbol for the new unit consists of the symbol for the prefix followed by the symbol for the base unit. Table 1-2 lists some common SI prefixes. For example, 1000 meters can be expressed as 1 kilometer or 1 km, and 0.01 meter can be expressed as 1 centimeter or 1 cm.

Symbols for Units and Quantities Symbols for SI units are printed in normal type. For example, m is the symbol for meter, and A is the symbol for ampere. Letter symbols are also used for the names of quantities in formulas. These symbols are printed in *italic* type. For example, *m* is the symbol for mass, and *A* is the symbol for area. Be careful not to confuse these different meanings of the same letters.

Dimensional Analysis Analyzing units can help in solving problems. The units on the left side of an equation must always be equivalent to the units on the right side of the equation. Quantities can be added or subtracted only if they have the same units. These facts can be used to check whether an answer is reasonable.

Table 1-2. Prefixes for Powers of 10		
Prefix	**Symbol**	**Notation**
tera-	T	10^{12}
giga-	G	10^{9}
mega-	M	10^{6}
kilo-	k	10^{3}
deci-	d	10^{-1}
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}

Formula for the period of a simple pendulum

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

Dimensional equation after substituting the units of acceleration due to gravity

$$T = \sqrt{\frac{\ell}{\frac{\ell}{T^2}}}$$

Figure 1-1. Analyze units to solve problems

For example, dimensional analysis for the period of a simple pendulum is shown in Figure 1-1. The time T required to complete one cycle of motion is the period of the pendulum, ℓ is the length, and g is the acceleration due to gravity. Because the period represents time, the expression on the right side of the equation must also have the dimension time. The units of the acceleration due to gravity, m/s^2, can be expressed as length ℓ in meters divided by T^2 in seconds squared, or $\frac{\ell}{T^2}$.

The units of length divide out and $T = \sqrt{T^2}$. The factor 2π has no units so it is not considered in the analysis.

Review Questions

1. Which term is not a fundamental unit?
   (1) kilogram
   (2) meter
   (3) second
   (4) watt

2. Which quantity and unit are correctly paired?
   (1) electric current — coulomb
   (2) frequency — hertz
   (3) power — joule
   (4) resistivity — ohm

3. Which amount of power is the *smallest*?
   (1) 1 gigawatt
   (2) 2 kilowatts
   (3) 3 megawatts
   (4) 4 watts

4. Which length is the *shortest*?
   (1) 1 μm (2) 2 mm (3) 3 nm (4) 4 cm

5. Continental drift speed is 1×10^{-9} meter per second. This is equivalent to a speed of
   (1) 1 Tm/s (2) 1 Gm/s (3) 1 nm/s (4) 1 pm/s

6. The diameter of 12-gauge wire is 2.053×10^{-3} meter. This is equivalent to 2.053
   (1) km (2) mm (3) μm (4) nm

7. The energy in half a tank of gasoline is 1,000,000,000 joules. Express this value in gigajoules.

8. The mean radius of Earth is 6,000,000 meters. Express this value in kilometers.

9. Which length is 10^6 times greater than a nanometer?
   (1) μm (2) mm (3) cm (4) km

10. The period of rotation of the Sun is 2.125×10^6 seconds. This is equivalent to 2.125
    (1) μs (2) ms (3) Ms (4) Ts

11. Human hair grows at the rate of 3 nanometers per second. This rate is equivalent to
    (1) 3×10^{-3} m/s
    (2) 3×10^{-6} m/s
    (3) 3×10^{-9} m/s
    (4) 3×10^{-12} m/s

12. The wavelength of red light is 7×10^{-7} meter. Express this value in nanometers.

13. If m represents mass in kg, v represents speed in m/s, and r represents radius in m, show that the force F in the formula $F = \frac{mv^2}{r}$ can be expressed in the unit $kg \cdot m/s^2$.

14. If PE_s represents the potential energy stored in a spring in $kg \cdot m^2/s^2$, and x represents the change in spring length from its equilibrium position in m, what is the unit for the spring constant k in the formula $PE_s = \frac{1}{2}kx^2$?

15. If F_e represents the electrostatic force in N that point charge q_1 in C exerts on point charge q_2 in C, and r represents the distance between the point charges in m, what is the unit for the electrostatic constant k in the formula $F = \frac{kq_1q_2}{r^2}$?
    (1) $N \cdot m^2/C^2$
    (2) $N \cdot m^2$
    (3) $N \cdot C^2/m^2$
    (4) $N \cdot m^2/C$

16. Using dimensional analysis, show that the expression $\frac{v^2}{d}$ has the same units as acceleration.

Tools for Measurement

In most laboratory investigations, you will make observations and measurements of physical quantities. You will be expected to select the appropriate piece of equipment, determine its scale, and make measurements to the proper number of significant figures.

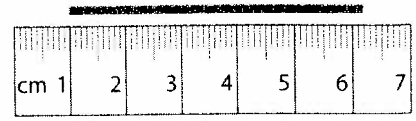


Figure 1-2. Metric ruler: The length of the wire is 5.20 cm.

Measuring Length

The length of an object or the total length of a path an object moves is measured with a metric ruler or meter stick. Path length is usually measured in meters, but occasionally centimeters are more appropriate. You can convert a measurement in centimeters to meters by dividing by 100. The piece of wire in Figure 1-2 has a length of 5.20 cm or 0.0520 m.

Figure 1-3. Electronic balance: The steel ball has a mass of 115.2 g.

Measuring Mass

The **mass,** or amount of matter contained in an object, can be measured with an electronic balance or a triple-beam balance. It is important that the balance be zeroed before determining the mass of an object.

The steel ball on the electronic balance in Figure 1-3 has a mass of 115.2 g or 0.1152 kg.

The block of wood on the triple-beam balance in Figure 1-4 has a mass of 208.50 g or 0.20850 kg. A mass that is determined in grams can be converted to kilograms by dividing by 1000.

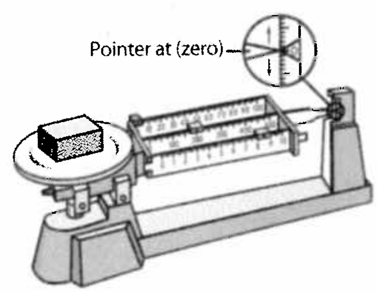


Figure 1-4. Triple-beam balance: The beam must be at zero when a reading of the mass is made.

Measuring Time

Elapsed time can be measured with a clock or stopwatch. As you know, one hour equals sixty minutes and one minute equals sixty seconds. Because many of the events you will be measuring in physics occur quickly, you may be asked to record elapsed time to the nearest hundredth of a second. The stopwatch in Figure 1-5 shows an elapsed time of 37.08 s.

Measuring Force

A push or pull on a mass is called a **force.** Forces are measured with a spring scale. Ranges on spring scales typically vary from 2.5 newtons to 20.0 newtons. Figure 1-6 shows a spring scale recording a force of 4.5 N as a block is lifted at constant speed.

Figure 1-5. Stopwatch: Minutes are recorded to the left of the colon. Seconds (to the one-hundredth place) are recorded to the right of the colon. The elapsed time is 37.08 s.

Measuring an Angle

A common unit for measuring angles is the degree (°), which is one ninetieth of a right angle. The protractor is an instrument used for measuring angles in degrees. Figure 1-7 shows a protractor being used to measure angle *AOB*. The wedge point of the protractor is on *O*, and the diameter of the semicircle lies on *OA*, one side of the angle. The other side of the angle intersects the semicircle at 47°. This reading gives the number of degrees in the angle. If the sides of the angle to be measured are too short to intersect the semicircle, they can be extended.

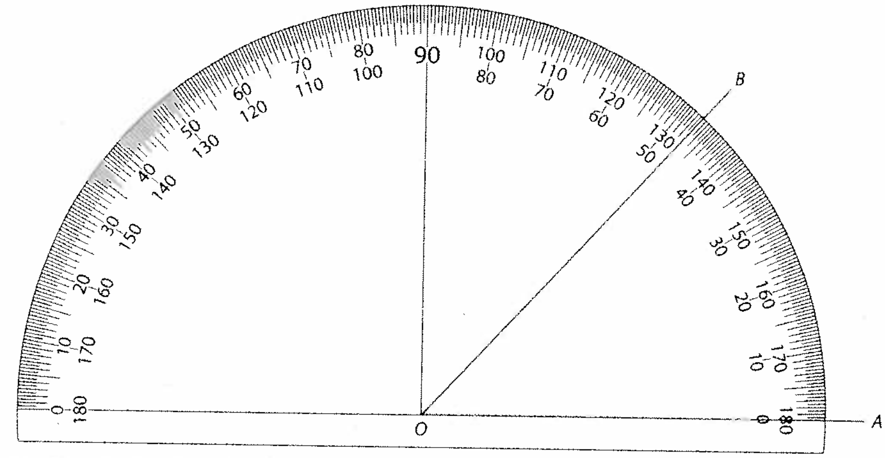


Figure 1-7. Protractor: Angle *AOB* has a measure of 47°.

Figure 1-6. Spring scale: Force or weight is measured with a spring scale. This scale reads 4.5 N.

Drawing an Angle To draw an angle of 25° with its vertex at point *P*, draw a line segment originating at *P*. Place the wedge point of the protractor on *P* and the diameter of the protractor semicircle along the line segment. Make a dot on the paper at the 25° mark on the inner set of degree readings. Draw a line from this point to *P*.

Trigonometry

The branch of mathematics that treats the relationships between the angles and sides of triangles is called trigonometry. Basic trigonometric relationships are used to solve some types of physics problems. Figure 1-8 shows a right triangle. Notice that side a is opposite angle θ, side b is adjacent to angle θ, and side c is the hypotenuse opposite the right angle.

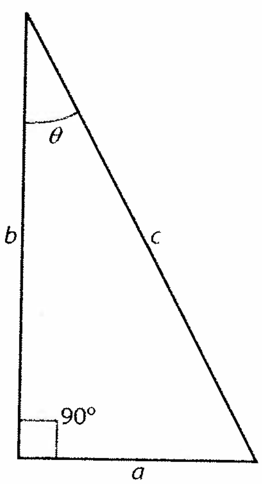


Figure 1-8. Right triangle

Important ratios of the sides of a right triangle in terms of angle θ include the following.

$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{b}{c}$$

$$\tan \theta = \frac{a}{b}$$

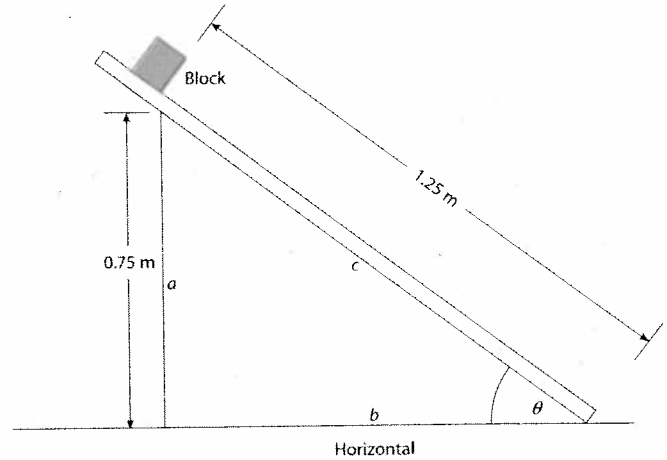
If the measure of angle θ is 30.°, the ratio of a to c is 0.50 because $\sin 30.° = 0.50$.

If you know the length of any two sides of a right triangle, you can find the length of the third side by using the Pythagorean theorem. The Pythagorean theorem is valid for right triangles only and has the following formula:

$$c^2 = a^2 + b^2$$

SAMPLE PROBLEM

A block is displaced a vertical distance of 0.75 meter as it slides down a 1.25-meter long plane inclined to the horizontal, as shown in the following diagram.
(a) Calculate the horizontal displacement of the block.
(b) Calculate the angle the plane makes with the horizontal.



SOLUTION: Relate the Pythagorean theorem to the diagram. Identify the known and unknown values.

Known	*Unknown*
$a = 0.75$ m	$b = ?$ m
$c = 1.25$ m	$\angle\theta = ?\,°$

1. Solve the formula for the Pythagorean theorem for the unknown, b.
$c^2 = a^2 + b^2$
$b = \sqrt{c^2 - a^2}$

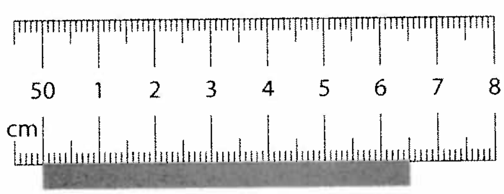
2. Substitute the known values and solve.
$b = \sqrt{(1.25\text{m})^2 - (0.75\text{ m})^2} = 1.0\text{ m}$

3. Write the formula for sin θ.
$\sin\theta = \dfrac{a}{c}$

4. Substitute the known values and solve for θ.
$\sin\theta = \dfrac{0.75\text{ m}}{1.25\text{ m}}$
$\theta = 37°$

Review Questions

17. A student measures a strip of metal using a metric ruler, as shown in the diagram below.



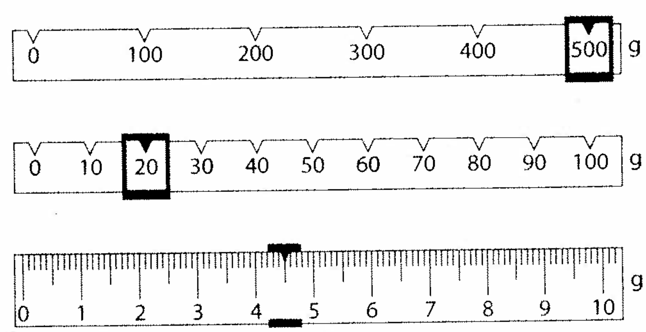
What is the length of the strip?
(1) 6.50 cm
(2) 56.5 mm
(3) 56.5 cm
(4) 5065 mm

18. The diagram below shows the cross-sectional area of a dowel.

Use a metric ruler to measure the diameter of the dowel to the nearest tenth of a centimeter.

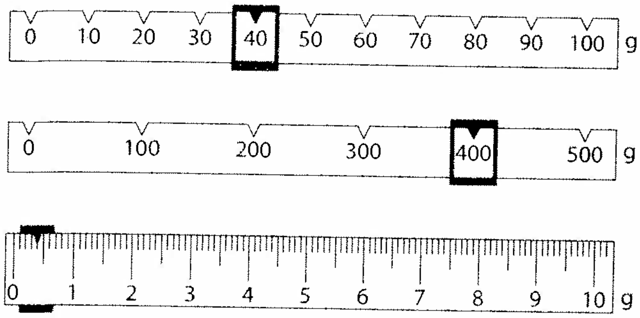
19. Express a length of 52.5 centimeters in meters.

20. The diagram below shows an enlarged view of the beams on a triple-beam balance.



What is the reading for the mass that is being measured?
(1) 251.0 g
(2) 524.5 g
(3) 5245 g
(4) 5,002,045 g

21. The diagram below shows an enlarged view of the beams of a triple-beam balance.



What is the reading, in kilograms, for the mass being measured?

22. The stopwatch below was used to time an event.

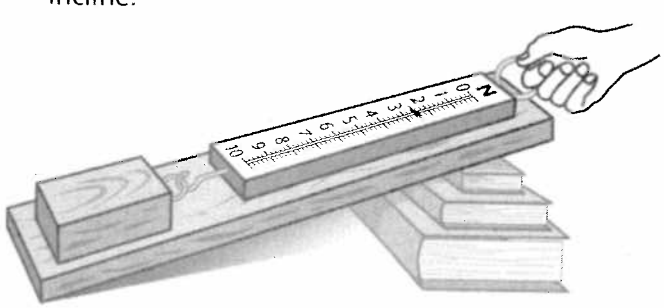
What is the elapsed time in seconds?

(1) 24.450 s
(2) 154.50 s
(3) 234.50 s
(4) 23,450 s

23. An electric lightbulb operates for 1 hour 15 minutes. What is the total time the light bulb operates in seconds?

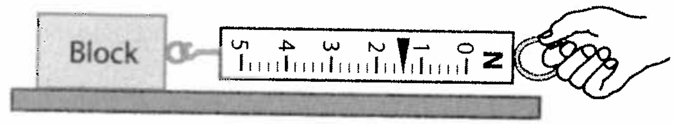
24. An electric iron is operated for 18 minutes at 120 volts. What is the total time the iron is operated in seconds?

25. The following diagram shows a spring scale being used to pull a wooden block up a wooden incline.



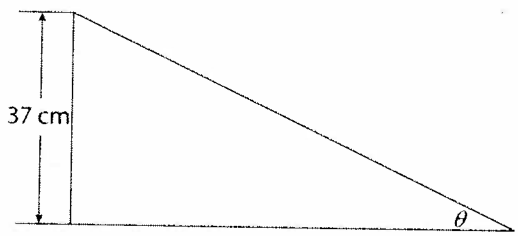
What is the magnitude of the force recorded on the spring scale?

26. The diagram below shows a spring scale attached to a wooden block as it is being pulled across a horizontal surface.



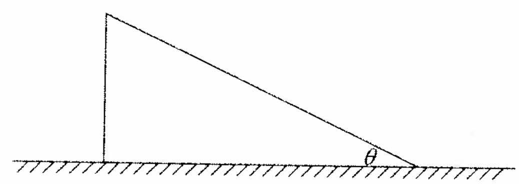
What is the magnitude of the force exerted on the spring scale?

27. The diagram below represents a ramp inclined to the horizontal at angle θ. The upper end of the ramp is 37 centimeters above the horizontal.



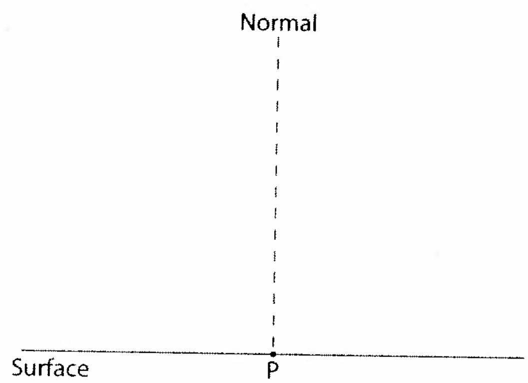
a) Using a protractor measure angle θ.
b) Calculate the length of the ramp.

28. The diagram below represents a ramp inclined at angle θ to the horizontal.

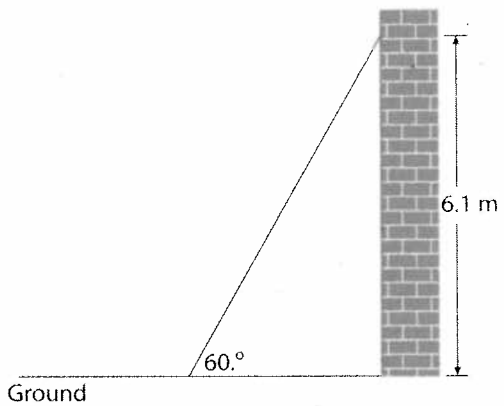


a) Using a protractor measure angle θ to the nearest degree.
b) What is $\sin\theta$?
c) What is $\cos\theta$?

29. On the diagram below, use a protractor and a straightedge to construct an angle of 40.° with the normal to the surface at point *P*.



30. The diagram below shows one end of a ladder resting against the side of a building 6.1 meters above the level ground. The other end of the ladder makes an angle of 60.° with the ground.

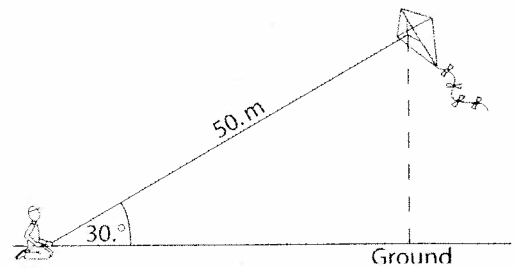


Using the scale in the drawing *or* a trigonometric function:

a) Calculate the length of the ladder.

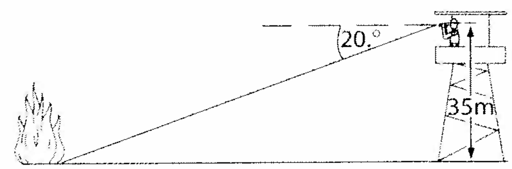
b) Calculate how far the base of the ladder is from the building.

31. A child flying a kite lets out 50. meters of string. The string makes an angle of 30.° with the ground, as shown in the diagram below.



Calculate the height of the kite above the ground.

32. A forest ranger, 35 meters above the ground in a tower, observes a blazing fire. The angle of depression to the base of the fire is 20.°, as shown in the diagram below.



Calculate the distance from the tower to the fire.

Uncertainty in Measurement

When a quantity is measured, the measurement always consists of some digits that are certain plus one digit whose value has been estimated. Thus, every measurement has an experimental uncertainty. The uncertainty can result from the quality and limitations of the measuring instrument, the skill of the person using the instrument, and the number of measurements made.

If several measurements taken of the same event are nearly identical, the measurements are said to be **precise.** If a measurement is very close to the accepted value found in a handbook, the measurement is said to be **accurate.** For example, the accepted value for the acceleration due to gravity near Earth's surface is 9.81 m/s^2. If a student measures this quantity as 9.98 m/s^2, 9.98 m/s^2, and 9.99 m/s^2, the measurements are precise, but not accurate.

Significant Figures (Significant Digits)

In a measured value, the digits that are known with certainty plus the one digit whose value has been estimated are called **significant figures** or significant digits. The greater the number of significant digits in a measurement, the greater the accuracy of the measurement.

Nonzero digits in a measurement are always significant. Zeroes appearing in a measurement may or may not be significant. The following rules should be applied *in order* to the zeroes in a measured value:

1. Zeroes that appear *before* a nonzero digit are *not* significant.
Examples: 0.002 m (1 significant figure) and 0.13 g (2 significant figures)

2. Zeroes that appear *between* nonzero digits are significant. Examples: 0. 705 kg (3 significant figures) and 2006 km (4 significant figures)
3. Zeroes that appear *after* a nonzero digit are significant *only* if (a) followed by a decimal point. Examples: 40 s (1 significant figure) and 20. m (2 significant figures); *or* if (b) they appear to the right of the decimal point. Examples: 37.0 cm (3 significant figures) and 4.100 m (4 significant figures)

A measurement of 0.040 900 kg utilizes all of the rules for zeroes and contains 5 significant digits.

If a whole-number measurement ends in two or more zeroes, it is not possible to indicate that some, but not all, of the zeroes are significant. For example, a measurement of 5200 m is interpreted to have only two significant figures, although it could actually represent a measurement to the nearest 10 meters. This situation is avoided by the use of scientific notation, which will be discussed later in this topic.

Addition and Subtraction with Measured Values

Measured values must have the same units before they are added or subtracted. For example, if the dimensions of a rectangle are recorded as 4.3 cm and 0.085 m, both measurements must be expressed either in centimeters or in meters before they can be combined by addition to find the perimeter of the rectangle. After adding or subtracting measured values expressed in the same units, the sum or difference is rounded to the same decimal place value as the least sensitive measurement.

Example A below shows that subtracting a measurement known to the nearest thousandth of a meter from a measurement known to the nearest tenth of a meter produces a difference known to the nearest tenth of a meter.

Similarly, in Example B below, adding measurements to the nearest hundredth of a centimeter, tenth of a centimeter, and centimeter produces a sum to the nearest centimeter.

Example A	Example B
	24.82 cm
31.1 m	4.7 cm
− 2.461 m	+ 2 cm
28.639 m = 28.6 m	31.52 cm = 32 cm

Multiplication and Division with Measured Values

When multiplying or dividing measured values, the operation is performed and the answer is rounded to the same number of significant figures as appears in the value having the lowest number of significant figures. In the example that follows, 2.6 cm has two significant figures, whereas 200.0 cm has four. Thus, the product of the two values can have only two significant figures.

$$(200.0 \text{ cm})(2.6 \text{ cm}) = 520 \text{ cm}^2$$

Notice that although both measurements are accurate to the nearest tenth of a centimeter, the last significant figure in the product is in the tens place. Thus, the product of a measurement with four significant figures and a measurement with two significant figures has only two significant figures.

Review Questions

33. A student measures the speed of yellow light in water to be 2.00×10^8 meters per second, 1.87×10^8 meters per second, and 2.39×10^8 meters per second. If the accepted value for the speed is 2.25×10^8 meters per second, the student's measurements are

(1) precise, only
(2) accurate, only
(3) both precise and accurate
(4) neither precise nor accurate

34. A student measures the length of a quarter-mile lap around the school's track to be 402.3 meters, 402.3 meters, and 402.5 meters. If the accepted value for the path length is 402.3 meters, the student's measurements are

(1) precise, only
(2) accurate, only
(3) both precise and accurate
(4) neither precise nor accurate

35. Which length measurement contains three significant figures?

(1) 0.203 m (3) 34.70 km
(2) 0.54 m (4) 570 cm

36. To what number of significant figures is a measurement of 14,020 grams expressed?

(1) 5 (2) 2 (3) 3 (4) 4

37. What is the area of a rectangle having dimensions of 9.8 meters and 12.7 meters?

(1) 100 m^2 (3) 124 m^2
(2) 120 m^2 (4) 124.46 m^2

38. Which mass measurement is expressed to two significant figures?

(1) 0.040 kg (3) 40 kg
(2) 0.4 kg (4) 405 kg

39. To what number of significant figures is a measurement of 7002 meters expressed?

(1) 1 (3) 3
(2) 2 (4) 4

40. A car travels 685 meters in 27 seconds. What is the average speed of the car?

(1) 25.370 m/s (3) 25.4 m/s
(2) 25.37 m/s (4) 25 m/s

41. A radar signal traveling at 3.00×10^8 meters per second is sent from Earth to the Moon and is received back at Earth in 2.56 seconds. What is the distance from Earth to the Moon?

(1) 7.68×10^8 m (3) 8×10^8 m
(2) 3.84×10^8 m (4) 4×10^8 m

42. What is the sum of 3.04 meters, 4.134 meters, and 6.1 meters?

43. What is the sum of 0.027 kilogram and 0.0023 kilogram?

44. To what number of significant figures is the measurement 0.705 meter expressed?

45. To what number of significant figures is the measurement 470 meters expressed?

46. Express forty meters with four significant figures.

47. Calculate the area of a rectangle having a length of 41.6 centimeters and a width of 2.3 centimeters.

48. Safety guidelines recommend an area of 5.6 $meters^2$ per student in a laboratory setting. Would a room having dimensions of 13.2 meters and 10.6 meters accommodate 24 students and comply with these guidelines? Justify your answer.

Scientific Notation

Measurements that have very large or very small values are usually expressed in **scientific notation.** Scientific notation consists of a number equal to or greater than one and less than ten followed by a multiplication sign and the base ten raised to some integral power. The general form of a number expressed in scientific notion is $A \times 10^n$. All of the digits in A are significant. For numbers having an absolute value greater than one, n is positive. For numbers having an absolute value less than one, n is negative. For a number having an absolute value of one, n is zero.
For example, the mean radius of Earth is 6,370,000 m or 6.37×10^6 m (3 significant figures). The universal gravitational constant is

0.000 000 000 066 7 $N \cdot m^2/kg^2$ or 6.67×10^{-11} $N \cdot m^2/kg^2$ (3 significant figures). The height of a physics student might be 1.75 meters or 1.75×10^0 meters (3 significant figures).

Addition and Subtraction Measurements written in scientific notation can be added or subtracted only if they are expressed in the same units and to the same power of ten. Sometimes, as in the example below, the power of ten must be changed first before adding or subtracting.

$$3.2 \times 10^2 \text{ m} + 4.73 \times 10^3 \text{ m} = 0.32 \times 10^3 \text{ m} + 4.73 \times 10^3 \text{ m} = 5.05 \times 10^3 \text{ m}$$

Multiplication and Division The commutative and associative laws for multiplication are used to find products and quotients of physical quantities written in scientific notation. Recall that the exponents are added when like bases are multiplied and the exponents are subtracted when like bases are divided. The general rule is as follows.

$$(A \times 10^n)(B \times 10^m) = (A \times B)(10^{n+m})$$

and

$$\frac{(A \times 10^n)}{(B \times 10^m)} = \frac{A}{B} \times 10^{n-m}$$

When multiplying and dividing measured values, the rules for significant figures apply to values expressed in scientific notation. Some examples follow.

$$(1.3 \times 10^5 \text{ m})(3.47 \times 10^2 \text{ m}) = 4.5 \times 10^7 \text{ m}^2$$

$$(1.3 \times 10^{-5} \text{ m})(3.47 \times 10^2 \text{ m}) = 4.5 \times 10^{-3} \text{ m}^2$$

$$(4.73 \times 10^5 \text{ m})(5.2 \times 10^2 \text{ m}) = 25 \times 10^7 \text{ m}^2 = 2.5 \times 10^8 \text{ m}^2$$

$$(8.4 \times 10^5 \text{ m}) \div (2.10 \times 10^2 \text{ m}) = 4.0 \times 10^3$$

$$(8.4 \times 10^5 \text{ m}) \div (2.10 \times 10^{-2} \text{ m}) = 4.0 \times 10^7$$

$$(2.10 \times 10^2 \text{ m}) \div (8.4 \times 10^5 \text{ m}) = 0.25 \times 10^{-3} = 2.5 \times 10^{-4}$$

SAMPLE PROBLEM A

Estimate the magnitude of the gravitational force that Earth exerts on the Moon and compare it with the actual value.

SOLUTION: Use the formula for the force due to gravity and the known values of the masses, the distance between centers, and the universal gravitational constant.

Known

$F_g = \dfrac{Gm_1m_2}{r^2}$

$G = 6.67 \times 10^{-11}$ $N \cdot m^2/kg^2$

$m_1 = 7.35 \times 10^{22}$ kg

$m_2 = 5.98 \times 10^{24}$ kg

$r = 3.84 \times 10^8$ m

Unknown

estimated F_g = ? N

1. Substitute the known values in the formula for the force due to gravity.

$$F_g = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.35 \times 10^{22} \text{ kg})(5.98 \times 10^{24} \text{ kg})}{(3.84 \times 10^8 \text{ m})^2}$$

2. Estimate the answer by rounding off each value to the nearest whole number and combining them.

$$F_g = \frac{(7 \times 10^{-11})(7 \times 10^{22})(6 \times 10^{24}) \text{ N}}{16 \times 10^{16}}$$

F_g (estimated) = 20×10^{19} N = 2×10^{20} N

3. Use a calculator to determine the actual magnitude of the force.

F_g (calculated) = 1.99×10^{20} N

The estimated value is close to the calculated value.

SAMPLE PROBLEM B

As the Voyager spacecraft passed the planet Uranus, it sent signals back to Earth. Determine the order of magnitude of the time in seconds for a signal to reach Earth. The distance from Earth to Uranus is 2.71×10^{12} meters. The speed of light in a vacuum is 3.00×10^8 meters per second.

SOLUTION: Identify the known and unknown values.

Known	*Unknown*
$d = 2.71 \times 10^{12}$ m	$t = ?$ s
$v = 3.00 \times 10^8$ m/s	

1. Round the known values to the nearest whole numbers and find the formula relating distance, time, and average speed.

$$\bar{v} = \frac{d}{t}$$

2. Solve for t. Substitute the rounded values and solve.

$$t = \frac{d}{\bar{v}} = \frac{3 \times 10^{12}\ \text{m}}{3 \times 10^8\ \text{m/s}} = 10^4\ \text{s}$$

The order of magnitude is 10^4.

Estimation and Orders of Magnitude

The technique of estimating the answer to a problem before performing the calculations makes it possible to quickly verify the procedures to be used and determine the reasonableness of the answer as in Sample Problem A. Estimating answers using orders of magnitude also helps in evaluating the reasonableness of an answer, as illustrated in Sample Problem B.

Review Questions

49. Express the diameter of a nickel, 0.021 meter, in scientific notation.

50. Express the mass of a car, 1500 kilograms, in scientific notation.

51. The jet engines of a 747 exert a force of 770,000 newtons. Express this value in scientific notation.

52. Divide $1.494\,57 \times 10^{11}$ meters, the average distance from the Sun to Earth, by 3.00×10^8 meters per second, the speed of light in a vacuum. Write your answer in scientific notation with the correct units and the appropriate number of significant figures.

53. What is the approximate width of a person's little finger?

(1) 1 m (2) 0.1 m (3) 0.01 m (4) 0.001 m

54. The length of a high school classroom is probably closest to

(1) 10^{-2} m (2) 10^{-1} m (3) 10^1 m (4) 10^4 m

55. The thickness of a dollar bill is closest to

(1) 1×10^{-4} m
(2) 1×10^{-2} m
(3) 1×10^{-1} m
(4) 1×10^1 m

56. Which measurement of an average classroom door is closest to 10^0 meter?

(1) thickness
(2) width
(3) height
(4) surface area

57. A flowerpot falls from a third-story window ledge to the ground. The total distance the flowerpot falls is closest to

(1) 10^0 m (2) 10^1 m (3) 10^2 m (4) 10^3 m

58. The approximate diameter of a 12-ounce can of root beer is

(1) 6.7×10^{-3} m
(2) 6.7×10^{-2} m
(3) 6.7×10^{-1} m
(4) 6.7×10^0 m

59. What is the approximate mass of a chicken egg?

(1) 1×10^1 kg
(2) 1×10^2 kg
(3) 1×10^{-1} kg
(4) 1×10^{-4} kg

60. A mass of one kilogram of nickels has a monetary value in United States dollars of approximately

(1) \$1.00
(2) \$0.10
(3) \$10.00
(4) \$1000.00

61. The mass of a physics textbook is closest to
(1) 10^3 kg (2) 10^1 kg (3) 10^0 kg (4) 10^{-2} kg

62. The mass of a high-school football player is approximately
(1) 10^0 kg (2) 10^1 kg (3) 10^2 kg (4) 10^3 kg

63. What is the approximate mass of an automobile?
(1) 10^1 kg (2) 10^2 kg (3) 10^3 kg (4) 10^6 kg

64. Approximately how many seconds are in three hours?
(1) 10^2 s (2) 10^3 s (3) 10^4 s (4) 10^5 s

65. The weight of an apple is closest to
(1) 10^{-2} N (2) 10^0 N (3) 10^2 N (4) 10^4 N

66. Which object weighs approximately 1 newton?
(1) dime
(2) paper clip
(3) physics student
(4) golf ball

67. The weight of a chicken egg is approximately
(1) 10^{-3} N (2) 10^{-2} N (3) 10^0 N (4) 10^2 N

68. The speed of a rifle bullet is 7×10^2 meters per second and the speed of a snail is 1×10^{-3} meter per second. How many times faster than the snail does the bullet travel?

69. The power of sunlight striking Earth is 1.7×10^{17} watts. How many 100-watt incandescent light bulbs would produce this amount of power?

Note: Use information found on the first page of the *Reference Tables for Physical Setting/Physics* in answering questions 70 through 73.

70. The acceleration due to gravity is approximately
(1) 10^{-1} m/s^2
(2) 10^0 m/s^2
(3) 10^1 m/s^2
(4) 10^3 m/s^2

71. What is the order of magnitude of the ratio of the charge on an electron to the mass of an electron?

72. What is the order of magnitude of the ratio of the speed of light in a vacuum to the speed of sound in air at STP?

73. What is the order of magnitude of the ratio of the mass of the Moon to the mass of Earth?

Evaluating Experimental Results

Experimental measurements made in any laboratory must be evaluated before they can be published in a scientific journal. An evaluation procedure has been developed for this purpose.

Data Analysis

In an experiment, for example to determine the relationship between the period of a simple pendulum and its length, multiple measurements are made of a given or identical event. Although there is a **range** of measurements or difference between the highest and lowest value in the data set, most of the measurements are close to the mean.

The notation $\sum_{i=1}^{n}$ is used to represent the sum of related terms. The index i is replaced by consecutive integers starting with the lower limit of summation written below the summation symbol Σ and ending with the upper limit of summation written above. Therefore, the **mean** or average $\bar{x}$ of a set of n measurements, where x_i is the individual measurement and f_i is the frequency of occurrence of that measurement, can be represented by Expression A below. The **variance** ν is the sum of the squares of the differences of the measurements from the mean, divided by the number of measurements, as shown in Expression B below. The **standard deviation** σ is the square root of the variance, shown by Expression C below.

Expression A

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i f_i}{\sum f_i}$$

Expression B

$$\nu = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{\sum f_i}$$

Expression C

$$\sigma = \sqrt{\nu} = \sqrt{\frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{\sum f_i}}$$

In a normal distribution, 68% of the data values lie between $\bar{x} - \sigma$ and $\bar{x} + \sigma$; 95% lie between $\bar{x} - 2\sigma$ and $\bar{x} + 2\sigma$; and 99.5% lie between $\bar{x} - 3\sigma$ and $\bar{x} + 3\sigma$.

SAMPLE PROBLEM

A student made seven measurements of the period of a simple pendulum of constant length released from the same point: 1.34 s, 1.28 s, 1.26 s, 1.28 s, 1.33 s, 1.33 s, and 1.28 s. Determine the range, mean, variance, and standard deviation for the data.

SOLUTION: Identify the known and unknown values.

Known	Unknown
$T_1 = 1.34$ s	range = ? s
$T_2 = 1.28$ s	mean $\bar{x}$ = ? s
$T_3 = 1.26$ s	variance ν = ? s^2
$T_4 = 1.28$ s	standard deviation σ = ? s
$T_5 = 1.33$ s	
$T_6 = 1.33$ s	
$T_7 = 1.28$ s	

1. Determine the range by subtracting the smallest measurement from the largest.
1.34 s − 1.26 s = 0.08 s

2. Set up a chart to simplify finding the mean, variance, and standard deviation.

3. Write in column 1 the four values of $T\,(x_i)$ that are different.

4. Write the frequency of each value of T in column 2.

5. Find the sum of the frequencies, Σf_i, and record it in column 2.
$\Sigma f_i = 1 + 3 + 2 + 1 = 7$

6. Multiply the values in column 1 (x_i) by the values in column 2 (f_i). Record the results in column 3.
$x_i f_i = (1.26\text{ s})(1) = 1.26\text{ s}$
$x_i f_i = (1.28\text{ s})(3) = 3.84\text{ s}$
$x_i f_i = (1.33\text{ s})(2) = 2.66\text{ s}$
$x_i f_i = (1.34\text{ s})(1) = 1.34\text{ s}$

7. Calculate the sum of all the $x_i f_i$ values, $\Sigma x_i f_i$, by adding the values in column 3.
$\Sigma x_i f_i = 1.26\text{ s} + 3.84\text{ s} + 2.66\text{ s} + 1.34\text{ s} = 9.10\text{ s}$

8. Determine the mean, $\bar{x}$, by dividing $\Sigma x_i f_i$ by Σf_i.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i f_i}{\Sigma f_i} = \frac{9.10\ s}{7} = 1.30\text{ s}$$

9. Subtract the mean from each value in column 1 and record the values in column 4.
1.26 s − 1.30 s = −0.04 s
1.28 s − 1.30 s = −0.02 s
1.33 s − 1.30 s = + 0.03 s
1.34 s − 1.30 s = + 0.04 s

10. Square the values in column 4 and record the results in column 5.
$(-0.04\text{ s})^2 = 0.0016\text{ s}^2$
$(-0.02\text{ s})^2 = 0.0004\text{ s}^2$
$(+0.03\text{ s})^2 = 0.0009\text{ s}^2$
$(-0.04\text{ s})^2 = 0.0016\text{ s}^2$

11. Multiply the values in column 5 by the frequencies in column 2. Record these products in column 6.
$(0.0016\text{ s}^2)(1) = 0.0016\text{ s}^2$
$(0.0004\text{ s}^2)(3) = 0.0012\text{ s}^2$
$(0.0009\text{ s}^2)(2) = 0.0018\text{ s}^2$
$(0.0016\text{ s}^2)(1) = 0.0016\text{ s}^2$

12. Add the values in column 6.
$\Sigma(x_i - \bar{x})^2 f_i = 0.0016\text{ s}^2 + 0.0012\text{ s}^2 + 0.0018\text{ s}^2 + 0.0016\text{ s}^2 = 0.0062\text{ s}^2$

13. Determine the variance using the following formula.

$$\nu = \frac{\sum_{i=1}^{4} f_i(x_i - \bar{x})^2}{\Sigma f_i} = \frac{0.0062\text{ s}^2}{7} = 8.9 \times 10^{-4}\text{ s}^2$$

14. Determine the standard deviation using the following formula.

$$\sigma = \sqrt{\nu} = \sqrt{\frac{\sum_{i=1}^{4} f_i(x_i - \bar{x})^2}{\Sigma f_i}}$$

$$= \sqrt{8.9 \times 10^{-4}\text{ s}^2} = 0.030\text{ s}$$

x_i (s)	f_i	$x_i f_i$ (s)	$x_i - \bar{x}$ (s)	$(x_i - \bar{x})^2$ (s^2)	$(x_i - \bar{x})^2 f_i$ (s^2)
1.26	1	1.26	−0.04	0.0016	0.0016
1.28	3	3.84	−0.02	0.0004	0.0012
1.33	2	2.66	+0.03	0.0009	0.0018
1.34	1	1.34	+0.04	0.0016	0.0016
	$\Sigma f_i = 7$	$\Sigma x_i f_i = 9.10$ s			$\Sigma(x_i - \bar{x})^2 f_i = 0.0062$ s^2

Percent Error

Measurements made during laboratory work may stand alone or be incorporated into one or more formulas to yield an **experimental value** for a physical quantity. In some instances, scientists have determined the most probable value or **accepted value** for quantities and published them in reference books. The difference between an experimental value and the published accepted value is called the **absolute error.** The **percent error** of a measurement can be calculated by dividing the absolute error by the accepted value and multiplying the quotient by 100.

$$\text{Percent error} = \frac{\text{absolute error}}{\text{accepted value}} \times 100$$

SAMPLE PROBLEM

In an experiment, a student determines that the acceleration due to gravity in the laboratory is 9.98 meters per second^2. Calculate the percent error. (According to the *Reference Tables for Physical Setting/Physics*, the accepted value for the acceleration due to gravity is 9.81 meters per second^2.)

SOLUTION: Identify the known and unknown values.

Known	*Unknown*
Experimental value of $g = 9.98$ m/s^2	Percent error = ? %
Accepted value of $g = 9.81$ m/s^2	

1. Determine the absolute error by finding the difference between the experimental measurement and the accepted value.

Absolute error $= 9.98\text{ m/s}^2 - 9.81\text{ m/s}^2$
$= 0.17\text{ m/s}^2$

2. Use the following formula to determine percent error.

$$\text{Percent error} = \frac{\text{absolute error}}{\text{accepted value}} \times 100$$

3. Substitute the known and calculated values and solve.

$$\text{Percent error} = \frac{0.17\text{ m/s}^2}{9.81\text{ m/s}^2} \times 100 = 1.7\%$$

Review Questions

74. In an experiment, a student measures the speed of sound in air to be 318 meters per second at STP. If the accepted value for the speed of sound under those conditions is 331 meters per second, what is the student's percent error?

(1) 3.9% (2) 3.93% (3) 4.09% (4) 4.1%

75. In an experiment, a student measures the speed of yellow light in water to be 2.00×10^8 meters per second. The accepted value for the speed is 2.25×10^8 meters per second. Calculate the student's percent error.

76. In an experiment a student obtained a value of 9.6 meters per second^2 for the acceleration due to gravity. The accepted value is 9.81 meters per second^2. Calculate the student's percent error.

Base your answers to questions 77 through 79 on the table below, which lists winning times to the nearest hundredth of a minute for the women's 400.-meter freestyle race at the Olympics.

Year	Time (min)
1960	4.66
1964	4.73
1968	4.51
1972	4.32
1976	4.17
1980	4.15
1984	4.12
1988	4.07

77. Find the range.

78. Determine the mean to the nearest hundredth of a minute.

79. Determine the standard deviation of these times to the nearest hundredth of a minute.

Base your answers to questions 80 through 82 on the data table below, which shows the frequency of the average daily temperatures during the month of June.

Temperature (°F)	Frequency
63	5
70.	3
78	4
79	3
80.	6
84	4
96	5

80. Find the range.

81. Determine the mean to the nearest tenth of a degree.

82. Determine the standard deviation to the nearest tenth of a degree.

Base your answers to questions 83 through 85 on the data table below, which shows the average snowfall in centimeters recorded one winter at a ski resort over a period of days.

Snowfall (cm)	Frequency
18	6
19	4
20.	4
21	3
24	5
26	3

83. Find the range.

84. Determine the mean to the nearest tenth of a centimeter.

85. Determine the standard deviation to the nearest tenth of a centimeter.

Graphing Data

The data collected in a physics experiment are often represented in graphical form. A graph makes it easier to determine whether there is a trend or pattern in the data.

Making a Graph

By convention, the **independent variable,** the one the experimenter changes, is graphed on the *x*- or horizontal axis. The **dependent variable,** the one that changes as a result of the changes made by the experimenter, is graphed on the *y*- or vertical axis. The axes are labeled with the quantities and their units are given in parenthesis. An appropriate, linear scale that accommodates the range of data is determined for each axis. It is not necessary to label every grid line. The graph should be titled as the dependent variable versus the independent variable. After the data points are plotted, a smooth line of best fit is drawn. The **line of best fit** is a straight or curved line which approximates the relationship among a set of data points. This line usually does not pass through all measured points. Sometimes the line of best fit is extrapolated. **Extrapolation** means extending the line beyond the region in which data was taken. This is important because the point where the extended line intersects the horizontal or vertical axis has physical significance.

The **slope,** or inclination of a graphed line, often has a physical meaning. On an *x*-*y* coordinate system, the slope of a line is defined as the ratio $\frac{\Delta y}{\Delta x}$ for any two points on the line. See Figure 1-9.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{\text{vertical change}}{\text{horizontal change}}$$

$$\text{slope} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

Figure 1-9. Slope defined

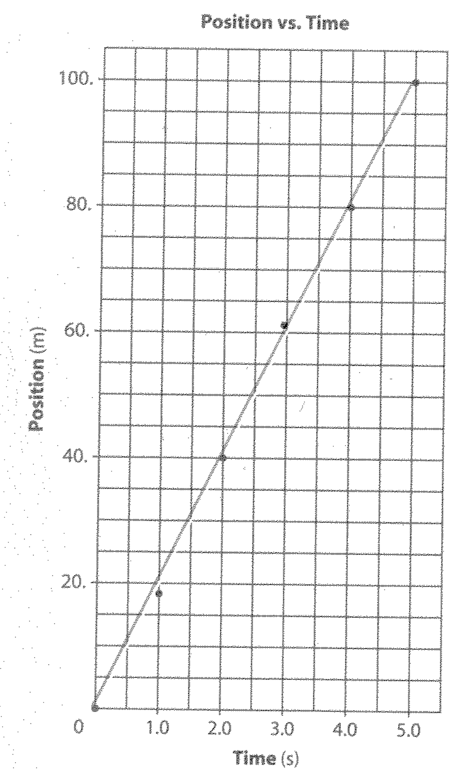
SAMPLE PROBLEM

The position of a moving car was measured at one-second intervals and recorded in the table.

Plot the data on the grid provided and draw the line of best fit. Calculate the slope of the line of best fit.

Time (s)	Position (m)
0.0	0
1.0	18
2.0	40.
3.0	62
4.0	80.
5.0	100.

SOLUTION: Using the information in the table, plot the data and draw the line of best fit.



Calculate the slope of the line.

$$\text{slope} = \frac{\text{change in position}}{\text{change in time}}$$

$$\text{slope} = \frac{100.\text{ m} - 40.\text{ m}}{5.0\text{ s} - 2.0\text{ s}}$$

slope = 20. m/s

In determining the slope of a graphed line, points directly from the data table can only be used if those points lie on the line of best fit. (**Note to student:** Although the formula for slope does not appear on the *Reference Tables for Physical Setting/ Physics*, calculating slope is testable.)

A horizontal line has a slope of zero. If a line is nearly horizontal, its slope has a small absolute value. If a line slants steeply, its slope has a large absolute value. A line that slopes downward to the right has a negative slope. Figure 1-10 illustrates some slopes of straight and curved lines.

Mathematical Relationships

Some of the common relationships that exist between quantities measured in physics are revealed by the shapes of graphs.

- Two quantities are **directly proportional** if an increase in one causes an increase in the other. The quotient of the quantities is a non-zero constant. The direct proportion $y = 2x$ or $\frac{y}{x} = 2$ is illustrated in Figure 1-11A.
- Two quantities are **inversely proportional** if an increase in one causes a decrease in the other. The product of the quantities is a non-zero constant. The equation $y = \frac{12}{x}$ or $xy = 12$ expresses the inverse proportion shown in Figure 1-11B.
- Two quantities have a **constant proportion** if an increase in one causes no change in the other. The equation $y = 6$, illustrated in Figure 1-11C, is a constant proportion.
- Two quantities have a **direct squared proportion** if an increase in one causes a squared increase in the other. The direct squared proportion $y = x^2$ is shown in Figure 1-11D.
- Two quantities have an **indirect squared proportion** if an increase in one causes a squared decrease in the other. The equation $y = \frac{12}{x^2}$ expresses the indirect squared proportion illustrated in Figure 1-11E.
- Figure 1-11F represents the equation $y = \sqrt{x}$.

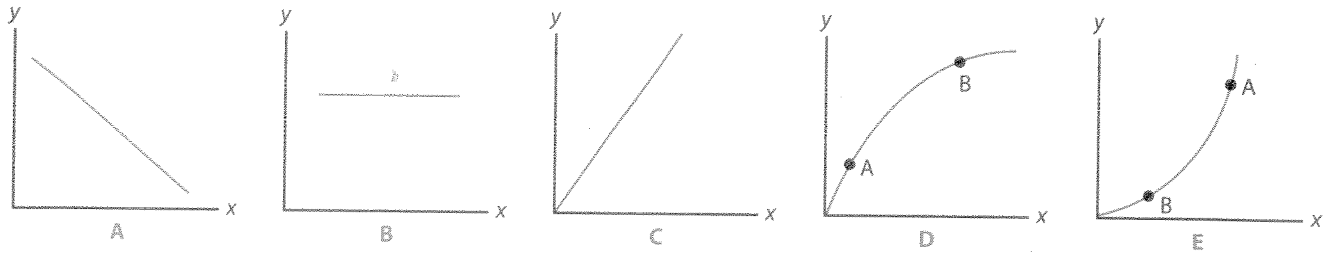


Figure 1-10. Slopes of common curves: The line in graph A has a negative slope. The line in graph B has a slope of zero. The line in graph C has a positive slope. In graphs D and E, the slope at point A is greater than at point B.

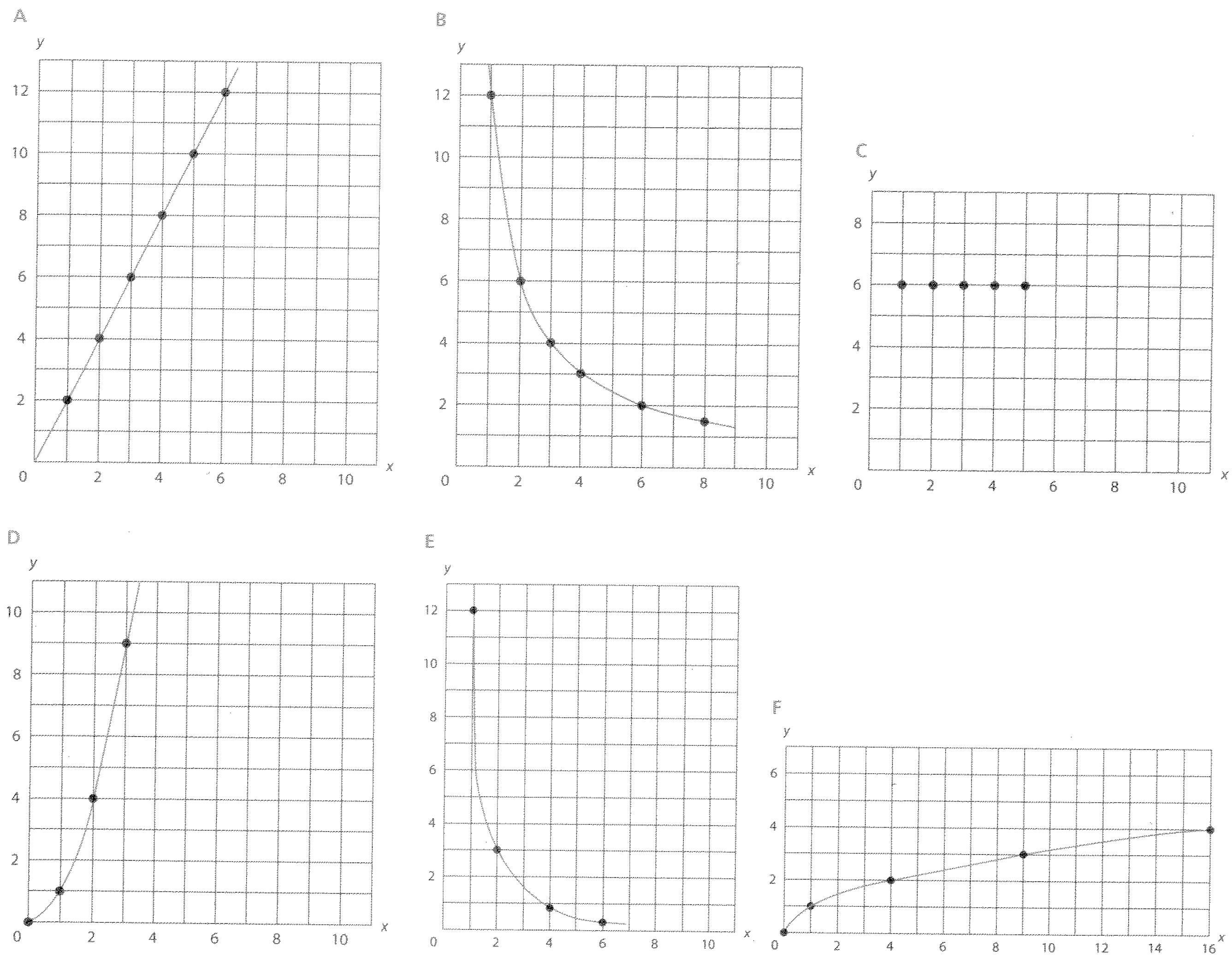
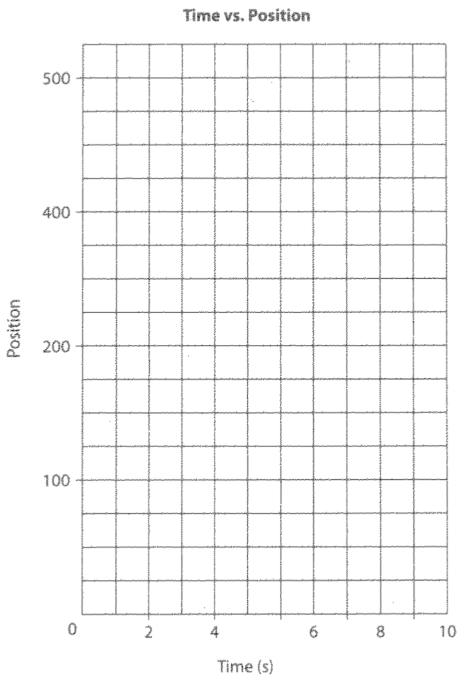


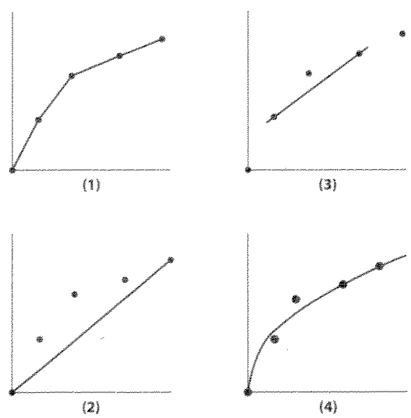
Figure 1-11. Graphs showing shapes of different proportions: (A) The graph of the direct proportion $y = 2x$ (B) The graph of the inverse proportion $y = \frac{12}{x}$ (C) The graph of the constant proportion $y = 6$ (D) The graph of the direct squared proportion $y = x^2$ (E) The graph of the indirect squared proportion $y = \frac{12}{x^2}$ (F) The graph of $y = \sqrt{x}$.

Review Questions

86. A student prepared the grid below to plot data collected in an experiment. List *four* errors the student made.



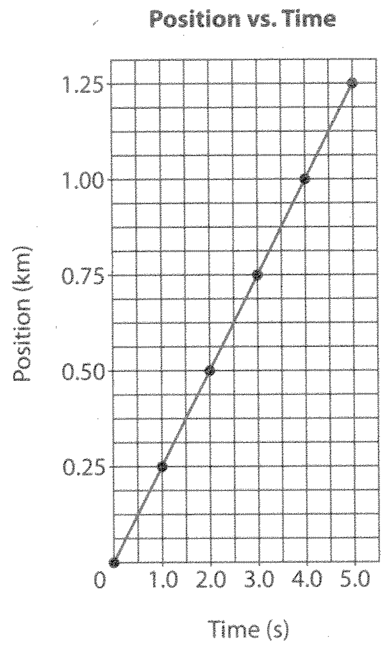
87. Which graph shows a properly drawn line of best fit?



88. A student varied the length of a simple pendulum and measured its period, which is the time required to complete one cycle of motion. In this experiment, time represents the variable that is

(1) dependent and graphed on the horizontal axis
(2) independent and graphed on the horizontal axis
(3) dependent and graphed on the vertical axis
(4) independent and graphed on the vertical axis

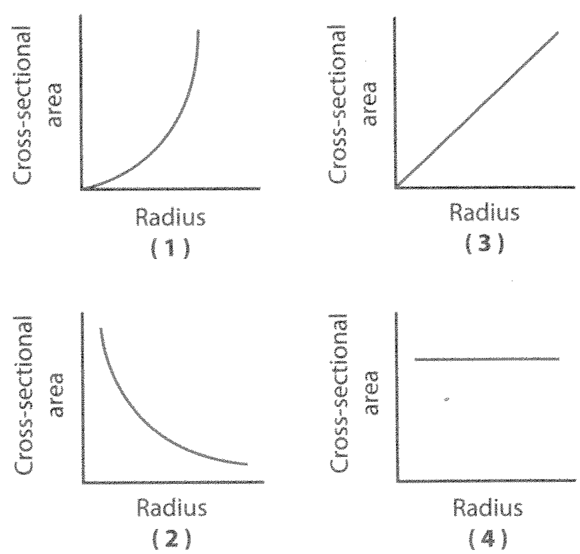
89. The graph below represents the motion of an object.



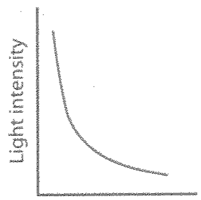
The slope of the line is

(1) 0.25 m/s
(2) 1.5 m/s
(3) 0.25 km/s
(4) 1.5 km/s

90. Which graph best represents the relationship between the cross-sectional area of a wire and its radius?



91. The graph that follows represents the relationship between light intensity and distance from a light source.



What kind of proportion exists between light intensity and distance?

(1) constant
(2) direct
(3) direct squared
(4) indirect squared

92. Sketch a graph that represents the relationship between the radius of a circle and its circumference.

93. According to Kepler's laws of planetary motion the ratio of the mean radius of the orbit of a planet cubed to the period of revolution of the planet squared is constant for all planets orbiting the Sun. Sketch a graph representing this relationship.

Scalar and Vector Quantities

Physical quantities can be categorized as either scalar or vector quantities. As physical quantities are introduced in this text, their scalar or vector nature will be indicated.

A **scalar** quantity has magnitude only, with no direction specified. Time and mass are scalar quantities. For example, 30 seconds and 45 kilograms are scalar quantities. The measurement of a scalar quantity is indicated by a number with an appropriate unit. Scalar quantities are added and subtracted according to the rules of arithmetic.

A **vector** quantity has both magnitude and direction. Velocity is a vector quantity because it must be described not only by a number with an appropriate unit, but also by a specified direction. For example, the velocity of a car might be described as 25 meters per second, due north. Vector quantities are added and subtracted using geometric or algebraic methods. These methods will be illustrated later in the text.

Solving Equations Using Algebra

Several axioms or statements are used in solving an equation for an unknown quantity. These axioms, which can be assumed to be true, include the following.

- If equals are added to equals, the sums are equal.
- If equals are subtracted from equals, the remainders are equal.
- If equals are multiplied by equals, the products are equal.
- If equals are divided by equals, the quotients are equal.
- A quantity may be substituted for its equal.
- Like powers or like roots of equals are equal.

You should make use of these axioms to isolate the unknown on the left side of an equation before substituting known values. Always include the units with the values in an equation. Although it is not necessary to align equal signs in the solution of an equation, it may help you keep your work orderly.

Mathematicians have agreed on the following order to be used in performing a series of operations:

1. Simplify the expression within each set of parentheses.
2. Perform exponents.
3. Perform the multiplications and divisions in order from left to right.
4. Do the additions and subtractions from left to right.

"*P*lease *e*xcuse *m*y *d*ear *A*unt *S*ue" is a useful memory device for this order: *p*arentheses, *e*xponents, *m*ultiplication and *d*ivision in order, and finally *a*ddition and *s*ubtraction in order.

Review Questions

94. Solve the following formulas for r.

(a) $F = \frac{mv^2}{r}$

(b) $A = \pi r^2$

(c) $C = 2\pi r$

(d) $F = G\frac{m_1 m_2}{r^2}$

95. Solve the following formulas for d.

(a) $\bar{v} = \frac{d}{t}$

(b) $P = \frac{Fd}{t}$

(c) $v_f{}^2 = v_i^2 + 2ad$

96. Solve the following formulas for v.

(a) $KE = \frac{1}{2}mv^2$

(b) $p = mv$

(c) $n = \frac{c}{v}$

97. Solve the following formulas for I.

(a) $R = \frac{V}{I}$

(b) $W = VIt$

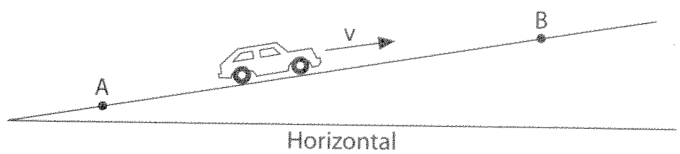
(c) $P = I^2R$

98. Express 1/299 792 458 second, the time it takes light to travel one meter in a vacuum, in scientific notation.

99. What is the approximate length of a baseball bat?

(1) 10^{-1} m
(2) 10^{0} m
(3) 10^{1} m
(4) 10^{2} m

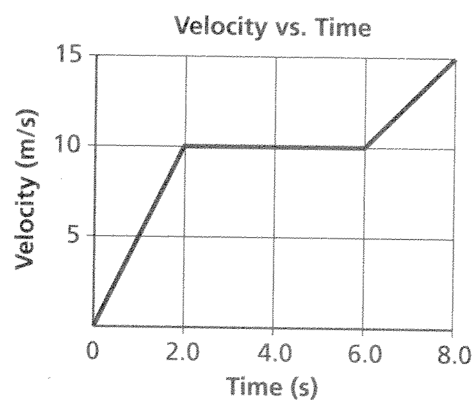
Base your answers to questions 100 and 101 on the diagram below, which represents a toy car traveling at constant speed *v* up an incline from point *A* to point *B*, a distance of 1.4 meters.



100. Determine the measure of the angle that the incline makes with the horizontal.

101. Calculate the vertical change in the car's position.

Base your answers to questions 102 and 103 on the graph below, which represents the velocity of an object traveling in a straight line as a function of time.



102. Calculate the total area under the curve.

103. Calculate the slope of the line in the time interval 6.0 seconds to 8.0 seconds.